國立臺北商業技術學院98學年度研究所碩士班考試入學試題

准考證號碼：□□□□□□（請考生自行填寫）

資訊科研所　　筆試科目：離散數學　　共 2 頁，第1頁

注意事項	
	1. 本科目合計100分，答錯不倒扣。
	2. 請於答案卷上依序作答，並標註清楚題號（含小題）。
	3. 考完請將答案卷及試題一併繳回。

► 1. (20%) True / False Questions:

(1) A path is a walk in which all edges are distinct.

(2) A cycle is a closed trail with no repeated vertices.

(3) In any graph with more than one vertex, there must exist two vertices of the same degree.

(4) A graph is bipartite if and only if it contains no odd cycle.

(5) Every subgraph of a tree is a tree.

(6) If G is a connected graph such that every two vertices of G are connected by a unique path, then G must be a tree.

(7) If A is the adjacency matrix of the complete graph on five vertices labeled by 1,2,3,4,5, then the $(5,3)$ entry of A^2 is 4.

(8) Let D be a digraph such that the indegree equals the outdegree for every vertex of G. Then D is Eulerian.

(9) If a relation is both symmetric and transitive, then it is reflexive.

(10) Define a relation $\sim$ on the set of all people in the world by $a \sim b$ if a and b were born in the same year. Then $\sim$ is an equivalence relation.

► 2. (10%) Which of the following sequence are graphical (i.e., there exists a simple graph whose degree sequence is the one specified)? In each case, either construct a graph, or explain why no graph exists.

(a) 3,3,2,2,1,1　(b) 6,5,5,4,3,3,3,2,2

► 3. (10%) Let $X = \{\{a\}, \{a,b\}, \{a,b,c\}, \{a,b,c,d\}, \{a,c\}, \{c,d\}\}$ and define $R = \{(U,V) : U, V \in X \text{ and } U \subseteq V\}$. Clearly, R is a partial ordering.

(a) Draw the Hasse diagram of the partial order.

(b) List all minimal, minimum, maximal, and maximum elements.

► 4. (10%) Solve the recursive formula:

$$T(n) = \begin{cases} 1 & \text{if } n \leqslant 1 \\ 2T(n/2) + n & \text{otherwise.} \end{cases}$$

背面尚有試題

▶ **5.** 選擇題（每題三分）

(1) How many items are there in the expansion of $(3x + 4y + 2z + 7w)^{10}$? (A) 286 (B) 334 (C) 122 (D) 684

(2) How many ways can one arrange four 1's and four -1's so that all eight partial sums (starting with the first summand) are nonnegative? (A) 88 (B) 64 (C) 32 (D) 14

(3) For any primitive statements p, q, r, any tautology T_0, and any contradiction F_0, which statement is wrong? (A) $(p \wedge q) \rightarrow r \Leftrightarrow \neg(p \wedge q) \vee r$ (B) $[p \wedge (p \rightarrow q)] \rightarrow q \Leftrightarrow T_0$ (C) $(p \vee q) \wedge \neg(\neg p \wedge q) \Leftrightarrow p \vee F_0$ (D) $\neg[[(p \vee q) \wedge r] \vee \neg q] \Leftrightarrow q \vee r$

(4) For a prescribed universe and any open statements $p(x)$, $q(x)$ in the variable x, which statement is wrong? (A) $\exists x\,[p(x) \wedge q(x)] \Leftrightarrow [\exists x\, p(x) \wedge \exists x\, q(x)]$ (B) $\exists x\,[p(x) \vee q(x)] \Leftrightarrow [\exists x\, p(x) \vee \exists x\, q(x)]$ (C) $\forall x\,[p(x) \wedge q(x)] \Leftrightarrow [\forall x\, p(x) \wedge \forall x\, q(x)]$ (D) $[\forall x\, p(x) \vee \forall x\, q(x)] \Rightarrow \forall x\,[p(x) \vee q(x)]$

(5) For any universe $\Re$ and any sets $A, B \subseteq \Re$, the *complement* of A is denoted by $\sim A = \Re - A$. Suppose that we define $A \Delta B = \{x \mid x \in A \cup B \wedge x \notin A \cap B\}$. Which following item is equivalent with $\sim(A \Delta B)$? (A) $\sim A \Delta \sim B$ (B) $A \Delta \sim B$ (C) $\sim A \cap B$ (D) $A \cup \sim B$

(6) Consider the following program segment, where i, j, and k are integer variables.

```
for i := 1 to 20 do
    for j := 1 to i do
        for k := 1 to j do
            print (i * j + k)
```

How many times is the print statement executed in this program segment? (A) 2644 (B) 6454 (C) 1540 (D) 3654

(7) For what base do we find that 251 + 445 = 1026? (A) 12 (B) 7 (C) 8 (D) 10

(8) For $a, b, c, d \in Z^+$, which statement is wrong? (A) if $a \geq b$, $\gcd(a, b) = \gcd(a-b, b)$ (B) the equation $ax + by = c$ has an integer solution x if and only if $\gcd(a, b)$ divides c (C) $cd = a$ and $\gcd(c, d) = b$ if and only if $b^2 | a$ (D) $\exists\, a, b\;\; a(a+1)(a+2) = b^2$

(9) Let A, B be finite sets and $|A| = 7$, $|B| = 4$. How many onto functions $f: A \rightarrow B$ can be occurred? (A) 8400 (B) 7640 (C) 3548 (D) 1286

(10) Let $A = \{a, b, c\}$. If function $f: A \times A \rightarrow A$, how many closed binary operations on A have b as the identity? (A) 27 (B) 48 (C) 81 (D) 96

▶ **6.** 證明題（每題十分）

(1) Please use the principle of mathematical induction to prove $5n < n^2 - 10$ for $n \geq 7$.

(2) For any $n \in Z^+$, prove that the integers $8n + 3$ and $5n + 2$ are relatively prime.

試題結束