國立臺北商業技術學院99學年度研究所碩士班考試入學試題

准考證號碼：□□□□□□（請考生自行填寫）

碩士班　　筆試科目：微積分　　共 2 頁，第 1 頁

注意事項	1. 本科目合計100分，答錯不倒扣。 2. 請於答案卷上依序作答，並標註清楚題號（含小題）。 3. 考完請將答案卷及試題一併繳回。

1. Integral can be valued using numerical methods of integration. Please use different numerical methods to approximate the value of a definite integral. Evaluate the integral $\int_1^5 \frac{1}{x}dx$, by dividing the interval [1,5] into 4 subintervals using

(a) Riemann sum [5 points]

(b) Midpoint rule [5 points]

(c) the Trapezoidal rule [5 points]

(d) the Simpson's rule [5 points]

2. Based on the change of variables: $x = r\cos\theta$, $y = r\sin\theta$, show the following partial differential equation [10 points]

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2}$$

3. (a) Solve the differential equation $\frac{dy}{dt} = ky(L-y)$ where $y>0$, $L-y>0$ and L is a constant. [10 points]

(b) Solve the differential equation $xy' - y = x$, $y(1) = 5$. [10 points]

4. $f(z,t) = \frac{1}{\sqrt{\pi a t}} e^{-\frac{(z-bt)^2}{at}}$

a. Find a and b such that $\frac{\partial f}{\partial t} = 0.03\frac{\partial f}{\partial z} + 0.02\frac{\partial^2 f}{\partial z^2}$ [10 points]

b. Find $\int_{-\infty}^{\infty} |z-bt| f(z,t)dz$ [10 points]

背面尚有試題

5. $f(z,t)=\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2t}}$ and $F(t)=\int_{-\sqrt{t}}^{\sqrt{t}}\frac{f(z,t)}{z}dz$ for t>0, find $\frac{dF}{dt}$. [10 points]

6. Find $\int_0^1\frac{x^7}{\sqrt{1-x^2}}dx$ [10 points]

7. Given $x=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}, x'=\begin{bmatrix}x_1 & x_2 & x_3\end{bmatrix}; \mu=\begin{bmatrix}10\\20\\30\end{bmatrix}; e=\begin{bmatrix}1\\1\\1\end{bmatrix}; A=\begin{bmatrix}100 & 100 & -50\\100 & 200 & 0\\-50 & 0 & 300\end{bmatrix}$, find the maximum of $x'Ax$ with constrains $x'\mu=20$ and $x'e=1$. [10 points]

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